

Target-searching on the percolation

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We study target-searching processes on a percolation, on which a hunter tracks a target by smelling odors it emits. The odor intensity is supposed to be inversely proportional to the distance it propagates. The Monte Carlo simulation is performed on a 2-dimensional bond-percolation above the threshold. Having no idea of the location of the target, the hunter determines its moves only by random attempts in each direction. For larger percolation connectivity $p \gtrsim 0.90$, it reveals a scaling law for the searching time versus the distance to the position of the target. The scaling exponent is dependent on the sensitivity of the hunter. For smaller p , the scaling law is broken and the probability of finding out the target significantly reduces. The hunter seems trapped in the cluster of the percolation and can hardly reach the goal.

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I. INTRODUCTION

In the past years, diffusion-controlled reactions have been extensively studied through random-walk models. Such applications range from chemical processes, electronic scavenging and recombination, to electronic and vibrational energy transfer in condensed media^{1,2,3,4,5,6,7}. Many works have been devoted to the target annihilation problem, in which randomly placed targets are annihilated by random walkers, and its dual of the trapping problem^{8,9}. Other models treat hindered diffusion problems which involve random point obstacles¹⁰. In these models, the tracer moves from site to site on a lattice and falls into wells of various depth at the sites. Another possibility is a mountain model, in which all sites are at zero energy and the barriers are on the bonds joining the sites¹¹. In the bond percolation problem, all sites are, in principle, accessible but the lattice is restricted by labelling the bonds which connect two sites as closed or open with probability p and $1 - p$, respectively¹². There exists a critical probability p_c (percolation threshold) such that an infinite conducting cluster exists for $p > p_c$ and does not exist for $p < p_c$. The percolation threshold depends on the geometric properties of the lattice and the kind of percolation problem to be studied (site or bond). For the bond percolation, $p_c = 0.5$ exactly in two dimensions and $p_c \approx 0.2488$ in three dimensions¹³.

In this paper, we present the target-searching game on a two-dimensional bond percolation. The active hunter is trying to find out a target which emits a special kind of odor. This kind of hunting processes, which frequently occurs in biological systems, such as a shark searching for foods by smelling the blood in the ocean, or honeybees flying in the countryside to locate the foraging-nectars^{14,15,16}, or in metabolic processes such as cell motions and chemotaxis^{17,18,19,20,21}, can be viewed as the target-oriented problems. The hunters try to reach the targets by following some kind of behavior rules. The structure of percolation can properly reflect the randomness of the environment.

The Monte Carlo simulations are performed on a 2-

dimensional bond percolation on an underlying square lattice. Since neither the distance nor the direction of the target is presumed to be known, the searcher should determine its moves by random attempts in each direction, just like a snake turns its head from side to side to test the variation of the odor intensity. There is some chance for the hunter to move in the wrong direction because of randomness. Hence it is not a traditional biased random-walk. After sorting each searching process in a time sequence by introducing a variable x to represent as a percentage, we find a scaling law for the searching time versus the distance to the position of the target for percolating probability $p \gtrsim 0.90$. The scaling exponent is found to be dependent on the sensitivity of the hunter. For $p \lesssim 0.90$, the scaling law is broken and the probability of reaching the goal reduces. The vacancy of some bonds frustrates the normal searching process. Under some circumstance, the hunter may be trapped in dead-end branches of the percolating cluster which it can hardly get out because of the constraint from the game rules.

The paper is arranged as following: The model is described in section II. The scaling property for larger bond connectivity is studied in section III and the trapping effect for smaller connectivity is studied in section IV. In section V, we discuss the trapping property and compare it with the trapping effect in biased random walks. A summary is included in section VI.

II. THE MODEL

In our Monte carlo experiment, we prepare a 3000×3000 underlying regular lattice and randomly remove a fraction $1 - p$ of the bonds. We restrict us to $p > p_c$ so that the network forms an infinite percolating cluster. The game rules are as follows: The hunter at the origin O is trying to find out a target somewhere which emits a special kind of odor. Since the hunter have no way to know the location of the target, it randomly moves around its present position to test the variation of the

odor intensity. z_0 is the present distance of the hunter to the target while z_1 is the corresponding distance of the next attempted step. Here distance is measured as the Euclidean distance. The Monte Carlo steps are implemented as: if $(z_0/z_1)^\alpha > \zeta$, where the parameter α reflects the sensitivity of the hunter and ζ is a random number, and there is a bond linking the two sites, then the attempt is accepted. Otherwise it is refused. This rule implies that the intensity of the signal emitted by the target is inversely proportional to the distance of the hunter to the target. Other choices of the relation do not alter the result qualitatively. By this way, the hunter approaches the goal in a stochastic style. When the hunter gets the goal, this round of search is over. The underlying lattice is large enough so that the hunter will not go beyond the edges of the percolation in the course of searching.

A total of 2000 simulations are performed for each given distance to the target. When a round of simulation is over, we record the searching time t . Then the time of all simulations are resorted increasingly by introducing a variable x to represent the percentage of searching times under a given value. When the hunter is far away from the goal, the ratio z_0/z_1 is close to 1. Most of the moving attempts are accepted, even the hunter walks in the wrong direction. The hunter appears to linger around for quite a while. Hence the motion of the hunter is nearly a Brownian random walk. As the goal is nearer, the ratio of z_0/z_1 gradually approaches 0.5 and the probability of being refused for the hunter moving in the wrong direction increases.

Figure 1(a) is a log-normal distribution of searching time t versus x for an initial distance $z = 137.1$ to the target. The sensitivity parameter $\alpha = 6$. Each curve represents a different bond-connected fraction p ranging from 0.60 to 0.97. In this figure, the curves are divided into two distinct regimes. In the smaller connectivity regime, the searching time t increases with reduction of p , indicating that the percolation frustrates the searching process. In the larger connectivity regime ($p \gtrsim 0.90$), the searching times almost keep invariant, indicating that small randomness can hardly affect the move of the hunter. Figure 1(b) displays the relation of the searching time and percolation connectivity p for $x = 50$. The smaller for p , the longer for searching time. Below we will study the properties of the two regimes.

III. THE SCALING REGIME

Figure 2(a) is a log-normal plot of the searching time t versus x for different initial distances at a percolation probability $p = 0.95$, with $z = 31.4, 65.6, 137.1, 188.4, 350.0$ from lower to upper and $\alpha = 6$. These curves are well-parallel to each other. Figure 2(b) shows linear relations of $\ln t$ and $\ln z$ at $x = 50$. From upper to lower, $\alpha = 6, 10, 14, 18, 22$, respectively. Hence each curve can be described by a single function

$f(x, \alpha)$, which reflects the statistical distribution of the searching time in the each simulation, plus a z -dependent function,

$$\ln t(x, z, \alpha) = f(x, \alpha) + \eta(\alpha) \ln z. \quad (1)$$

In Fig. 3 we study the dependence of the searching-time with respect to the sensitivity parameter α . Figure 3(a) shows the curves for $\alpha = 6, 10, 14, 18, 22$, respectively. After properly rescaling the curves in (a) by times $\ln t$ with a coefficient α^β , all the curves become parallel (Fig.3(b)). From Fig.3(b), we deduce $\alpha^\beta \ln t = \tilde{f}(x) + \tilde{\phi}(z, \alpha)$, where $\beta = 0.623$. By comparing with Eq.(1), one gets

$$\ln t(x, z, \alpha) = \alpha^{-\beta} f(x) + \eta(\alpha) \ln z. \quad (2)$$

To go further, we probe the relation of searching with parameter α , as shown in Fig.4. It reveals a good linear relation $\ln(\alpha^\beta \ln t) \propto \gamma \ln \alpha$, with the slope $\gamma = 0.5$.

Combining all the above factors, we can figure out a scaling law for the searching-time with respect to the distance as well as sensitivity parameter α ,

$$\ln t = \alpha^{-\beta} f(x) + \alpha^{\gamma-\beta} \ln z, \quad (3)$$

or

$$t(x, z, \alpha) \propto e^{f(x)/\alpha^\beta} \cdot z^\delta. \quad (4)$$

The scaling exponent is found to be $\delta = \alpha^{\gamma-\beta}$. It shows that the more sensitive the hunter, the less time it costs to reach the target.

IV. THE TRAPPING REGIME

Now we turn to study the properties of the regime $p < 0.90$. As shown in Fig.1, the searching processes for $p < 0.90$ are greatly affected by the absence of the bonds, with larger searching time for smaller fraction of bond-connectivity p . This means that strong disorders tend to block the searching process of the hunter. Furthermore, the scaling law revealed in section III is broken.

Figure 5 shows the crossover from the scaling regime to the trapping regime. The search processes are averaged over 200 configurations of percolation. Given the upper limit of walking steps t_c , above which one can consider that the hunter fails to reach the target. The vertical axis in Fig.5 is the probability of success to find out the target within the time period t_c . In Fig.5(a), the initial distance $z = 137.1$ and $\alpha = 6, 10, 14, 18$ from upper to lower. $t_c = 1,000,000$ steps for all simulations. It can be seen that for $p > 0.90$, the hunter always reaches the goal in this time limit. As p decreases, the probability of missing the goal increases significantly. It is notable that the more sensitive, the more probable for the hunter to fail to reach the goal. This trend is just reverse with that in the scaling regime. Under some circumstance, the hunter will never reach the goal because

the searching time goes to infinity. The hunter seems trapped in some dead-end branches of the percolation, although the percolation is still well-connected. We call this range of connectivity p the trapping regime. Figure 5(b) is the probability of success for three different time limits $t_c = 100, 000, 500, 000, 1, 000, 000$, with $\alpha = 6$. Not surprisingly, the probability of success increases with the upper limit of walking time t_c .

V. DISCUSSION

It should be mentioned that the trapping of the hunter in the homogenous percolating cluster is somehow analogous to the trapping of a random walker in a strong biased diffusion. When the biased field B exceeds a critical value B_c , the walkers may enter a dead-end branch so that it is difficult to escape away by overcoming the biased field^{22,23,24}. The drift velocity tends to zero and the particle will never reach the opposite end. This is easily seen: For a trap of depth l the potential barrier to cross increases with l , and the trapping time varies as $(\frac{1+B}{1-B})^l$. The density $\rho(l)$ of traps of depth l varies as $\exp(-l/\xi)$, where ξ is the p -dependent percolation correlation length of the system. Hence the average trapping time per step along the backbone is

$$\sum_{l=1}^{\infty} \rho(l) \left(\frac{1+B}{1-B}\right)^l \quad (5)$$

This summation converges only for $B < B_c = \tanh(\frac{1}{2\xi})$. For $B > B_c$, the asymptotic velocity is zero, but the mean displacement of the particle $\langle z \rangle$ increases as t^a with $a < 1$. In a time t , the particle can, on the average, only travel a distance t^a before it encounters a trap with trapping time bigger than t , and gets stuck there. Eventually, it will exit from this trap, only to get stuck in other traps. In our case, however, the constrain come from the hunter trying to trace the target. When it unfortunately enters a series of dead-end branches of the infinite cluster, it will be difficult to get out of them. In the biased diffusion processes, the larger biased field leads to more strict trapping. In the hunting processes, increase α will also enforce the trapping effect.

VI. SUMMARY

We have studied the searching processes on a percolation. It is found that when de disorders are low, there is a scaling law between the searching time and the distance to the target. For strong disordered systems, the scaling law is broken. The hunter may be trapped in the dead-end branches and can hardly reach the target. This trapping effect is somehow analogous to the trapping in the biased random walk processes. Our study may be instructive for zoologists or entomologists when they explore the food-foraging behavior of wild animals.

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Figure Captions

Figure 1 (a) Time-sorted curves for 2000 simulations. The horizontal axis x is the sequence of searching time represented as a percentage and the vertical axis is logarithmic time. $\alpha = 6$. From upper to lower, $p = 0.6, 0.7, 0.8, 0.9, 0.93, 0.95, 0.97$. As $p \gtrsim 0.90$, the curves almost fall into one. (b) p -dependence of searching time for $x = 50$.

Figure 2 (a) Time-sorted curves at $p = 0.95$ for various distances z from the origin, with $z = 31.4, 65.6, 137.1, 188.4$ from bottom to top. These curves are parallel to each other. (b) A plot of $\ln t$ versus $\ln z$ at $x = 50$ for various sensitivity parameter. From upper to lower, $\alpha = 6, 10, 14, 18, 22$, respectively. It reveals a

linear relation.

Figure 3 Time-sorted curves for 2000 simulations for various parameter values α . The horizontal axis x is the sequence of searching times represented as a percentage and the vertical axis is logarithmic time. The original distance is fixed at $z = 137.1$. (a) is for $\alpha = 6, 10, 14, 18, 22$ from top to bottom. (b) shows the rescaled curves of (a) for $\alpha = 6, 10, 14, 18, 22$ from bottom to top. These curves are parallel to each other.

Figure 4 A linear relation of $\ln(\alpha^\beta \ln t)$ vs $\ln \alpha$ at $x = 50$. The distance $z = 137.1$. The slope is $\gamma = 0.5$.

Figure 5 (a) Crossover from the scaling regime to trapping regime. The vertical axis is the probability of success of finding out the goal at a given time period $t_c = 1,000,000$. The horizontal axis is the bond-connected fraction. The original distance $z = 137.1$. (b) The same as in (a) for various time limits. The sensitivity parameter is fixed at $\alpha = 6$.









